FINAL REPORT

"Miniature Thermoacoustic Refrigerator"

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THE UNIVERSITY OF UTAH

May 6, 1994

Ms. June A. Hawley
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Reference: ONR 244:JAH:hs
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Dear Ms. Hawley,

Please find enclosed the final report for grant N00014-91-J-2015, which expired November 10, 1993.

I am sorry for the delay.

Sincerely,

Orest Symko

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TABLE OF CONTENTS

Abstract

A miniature thermoacoustic refrigerator was developed for cooling electronic components. The refrigerator is driven at 5,000 Hz by a piezoelectric driver and it consists of an acoustic resonator, a stack, and copper heat exchangers. The working fluid is pressurized helium gas between 10 to 20 atmospheres. Sound was produced with different piezoelectric drivers, a monomorph, a bimorph, and Tonpilz driver to achieve the highest intensity possible. Since the operating frequency is high, the refrigerator is compact, being about 10 cm long. Temperature differences of 12°C across the stack were achieved at sound levels of 160dB. The performance of the refrigerator scaled with frequency according to existing models of thermoacoustic heat engines.

1. Goals

This project deals with the development of a thermoacoustic refrigerator which operates at a frequency which is approximately ten times higher than in previously achieved thermoacoustic refrigerators. By operating in the frequency range of 5,000 Hz, the geometrical requirements will make the refrigerator small, compact, simple, and suitable for the refrigeration of small devices and/or electronic circuits. Operation at high frequencies raises questions of efficiency, performance, materials, compactness, and fundamental limitations which deserve to be explored.

Specifically the goals of this project were:

-- design a thermoacoustic refrigerator which will operate at 5,000 Hz
-- use piezoelectric drivers
-- demonstrate refrigeration in a unit operating at 5,000 Hz
-- evaluate its performance
-- assess its potentials as a refrigerator

Because of the strong frequency dependence of many parameters in thermoacoustic refrigeration, a tenfold increase in frequency over the conventional refrigerators brings in a variety of interesting problems and raises fundamental questions.

2. Background

In 1975, Merkli and Thomann[1] demonstrated that refrigeration can occur by the thermoacoustic effect in a simple tube driven acoustically at approximately 120 Hz. Later Wheatley and collaborators[2] demonstrated that substantial refrigeration occurs when an internal structure, a plastic stack, is added to an acoustically driven resonant tube. Using a high intensity frequency of 515 Hz, they achieved over 100°C temperature change in a unit which was over 55 cm long. Their unit was driven by a hi-fi midrange loudspeaker which uses a strong permanent magnet to interact with the voice coil. The sound field was further amplified by a Helmholtz resonator. This type of refrigerator is based on the thermal interaction of a sound field with a stack of plastic plates; this is an irreversible process. To produce cooling certain conditions have to be met, in particular it is important that the timing is correct, i.e.

$$\omega \tau \approx 1 \tag{1}$$

Here τ is the thermal time constant for energy exchange by thermal diffusion between the working fluid, the gas in the resonator, and the plates; ω is the angular frequency of the sound field. When the frequency is

$$\omega \tau >> 1 \tag{2}$$

the process is adiabatic and nothing will happen. In fact, it is too high a frequency for proper operation of the refrigerator. On the other hand when

$$\omega \tau << 1 \tag{3}$$

the frequency is so low that the process is isothermal and here as well there is no cooling.

When the acoustic field interacts thermally well enough with the stack, heat pumping action takes place. There is energy exchange between the working fluid and the the stack of plates within a thermal penetration depth δ_k given by

$$\delta_k \equiv (2\kappa/\omega)^{1/2} \tag{4}$$

where κ is the thermal diffusivity (equal to $K/\rho C_p$, with K being the thermal conductivity, C_p its specific heat and ρ the density of the fluid). Because the thermoacoustic refrigerator here is based on an irreversible phenomenon, heat conduction between the sound field and the plates, the refrigeration is expected to be less efficient than in a Carnot cycle. However it is an important refrigerator since it is simple and it has no moving parts other than the acoustic driver.

The basic requirements for such a refrigerator are a source of sound, a resonator to increase the sound level, a stack of plates for maintaining the correct phasing between sound pressure and its velocity, a set of heat exchangers to establish the flow of heat to and from heat reservoirs, and a working fluid, the gas.

On resonance an acoustic resonator will provide pressure amplification of the driving pressure P_o according to:

$$P = QP_o \tag{5}$$

where Q is the quality factor of the resonator. It is given by

$$Q = \omega M/R \tag{6}$$

at an angular frequency ω for an acoustic mass M. Here R is the acoustic resistance and it consists of viscous losses as well as radiation losses. The frictional losses consist of two contributions[3]:

(i) Wall effects due to friction and the exchange of heat energy between the fluid and the wall. This is expressed as a wall absorption coefficient α of an acoustic wave propagating in air at 20°C and in a circular tube of radius a:

$$\alpha = (2.93 \times 10^{-5} f^{1/2})/ a \tag{7}$$

(ii) Effects in the medium itself due to viscous losses, heat conduction losses, and losses due to molecular exchange of energy. Their contributions appear in the absorption coefficient α given by

$$\alpha = 0.54 \times 10^{-11} f^2 \tag{8}$$

for helium at 20°C and one atmosphere

and

$$\alpha = 1.5 \times 10^{-11} f^2 \tag{9}$$

for air at 20°C and at one atmosphere

Effects of medium losses will start to become important around 1 MHz.

At low frequencies the quality factor Q of the resonator is given by:

$$Q = \omega c/2\alpha \tag{10}$$

i.e. it varies as $\omega^{1/2}$, at low frequencies.

In choosing the acoustic driver various criteria are used. The main one is the need for a large sound output so as to drive the resonator. Efficiency is important. Calculations have been made for a transversely oscillating sphere of radius a_1. The time average of the acoustic power emitted by a sphere is:

$$P_{av} = \left(\frac{(ka_1)^4}{4+(ka_1)^4} \right) \rho c (v^2/3)_{av}\ 4\pi a^2 \tag{11}$$

where v is the speed of the sphere, ρc the acoustic impedance of the medium and k is the wave number. A disk of radius a will behave similarly (in the low frequency limit $ka<<1$) when a_1 in Eq. 11 is replaced by 0.715a. The power radiated then varies as ω^4. A tweeter (which is small) in a loudspeaker usually radiates more sound power than a woofer (which is larger!).

The working fluid for the refrigerator should be a gas whose thermal properties will provide optimum performance of the refrigerator. Desirable properties are small viscosity, high thermal conductivity, and ideal gas behavior[4]. Table I gives some important quantities of gases which could be useful for thermoacoustic refrigeration.

Table I. Properties of some gases

	^{4}Helium	^{3}Helium	Helium (89%)-Xenon (11%)	Air
ρ (kgm/m^3)	1.9×10^{-1}	1.42×10^{-1}	16.2	1.21
K (w/cm/deg)	1.5×10^{-3}	1.73×10^{-3}	10^{-3}	2.6×10^{-4}
C_p (J/gm κ)	5.2		1.145	1.00
c (m/sec)	1008	1167	456.6	344 (20°C)
ρc (gm/sec/cm^2)	19	16.6	736	41.4
η (μ poise)	19.41 (1 atm)	160.85	236	185 (1 atm)
Pr	0.68	0.67	0.27	0.72

Here ρ is the density, k the thermal conductivity, c_p the specific heat, c the speed of sound, ρc the impedance, η the viscosity, and Pr the Prandl number. In this project helium gas was used, primarily so that a comparison could be made with the early data of Wheatley and collaborators. The merits of other gases will be discussed later. Data for the helium xenon gas mixture were taken from ref. 5.

We chose the operating frequency of 4,000 - 5,000 Hz arbitrarily. It is a convenient frequency range as hi-fi equipment is cheap and easily available. As a first approximation we assumed that the operation at 5,000 Hz would scale with the original 500 Hz refrigerator of Wheatley and collaborators.

3. Experimental Details

The basic apparatus consists of a piezoelectric driver, an acoustic resonator, a stack of plates, heat exchangers, and helium gas. Temperatures were monitored by a constantan-chromium thermocouple. The working fluid was helium gas which was pressurized from 10 to 20 atmospheres. As the refrigerator is small, its total area is also small and hence higher pressures could be used without exceeding the strength limit of the materials. Fig. 1 shows the basic thermoacoustic refrigerator studied here. Its parts will now be discussed.

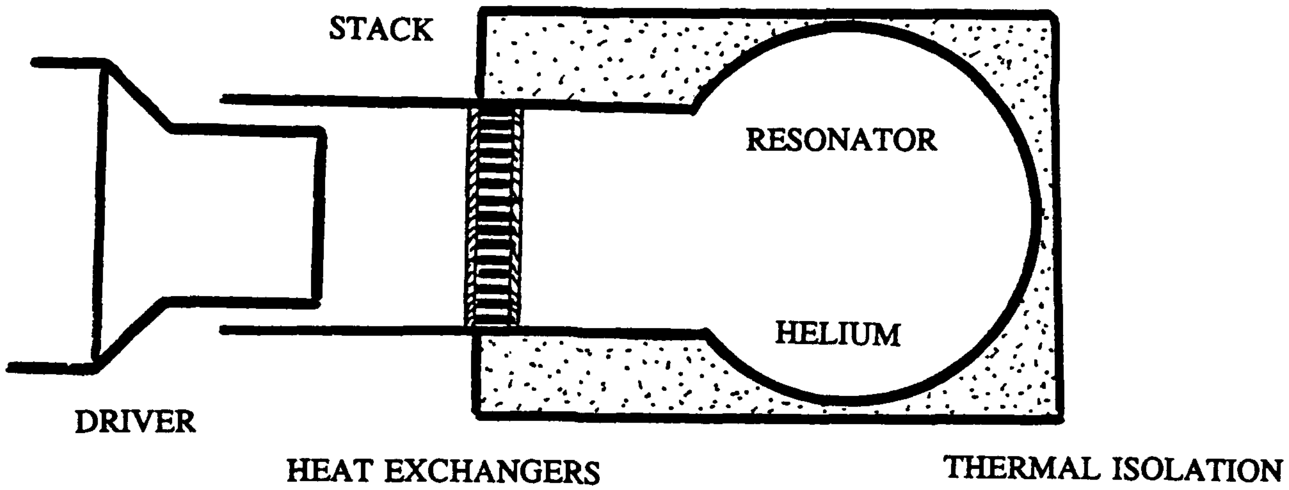

Fig. 1 High frequency thermoacoustic refrigerator

A. <u>Driver</u>

The drivers that were studied were all piezoelectric type of drivers. Piezoelectric drivers were used because:

--they can be very efficient; they can go up to 90% efficiency, depending on the load.

--they are compact and light (this is important when the payload has to be small). No magnets are needed as in the regular electro-magnetic drivers.

--the losses are small

--they can produce high sound intensities

The power dissipated by a piezoelectric element with capacitance C, driven at an rms voltage V, and frequency f is:

$$P = V^2 / R_e \tag{12}$$

where $R_e = (6.28\, f\, C \tan\delta)^{-1}$

For material such as lead zirconate titanate, type EC66, the dissipation factor ($\tan\delta$) is 0.07. The sound intensity produced by a piezoelectric driver can be high, of the order of ~1w/cm^2, for air loading.

Fig. 2 shows the drivers that were used, in this project a monomorph, a bimorph, and a Tonpilz driver.

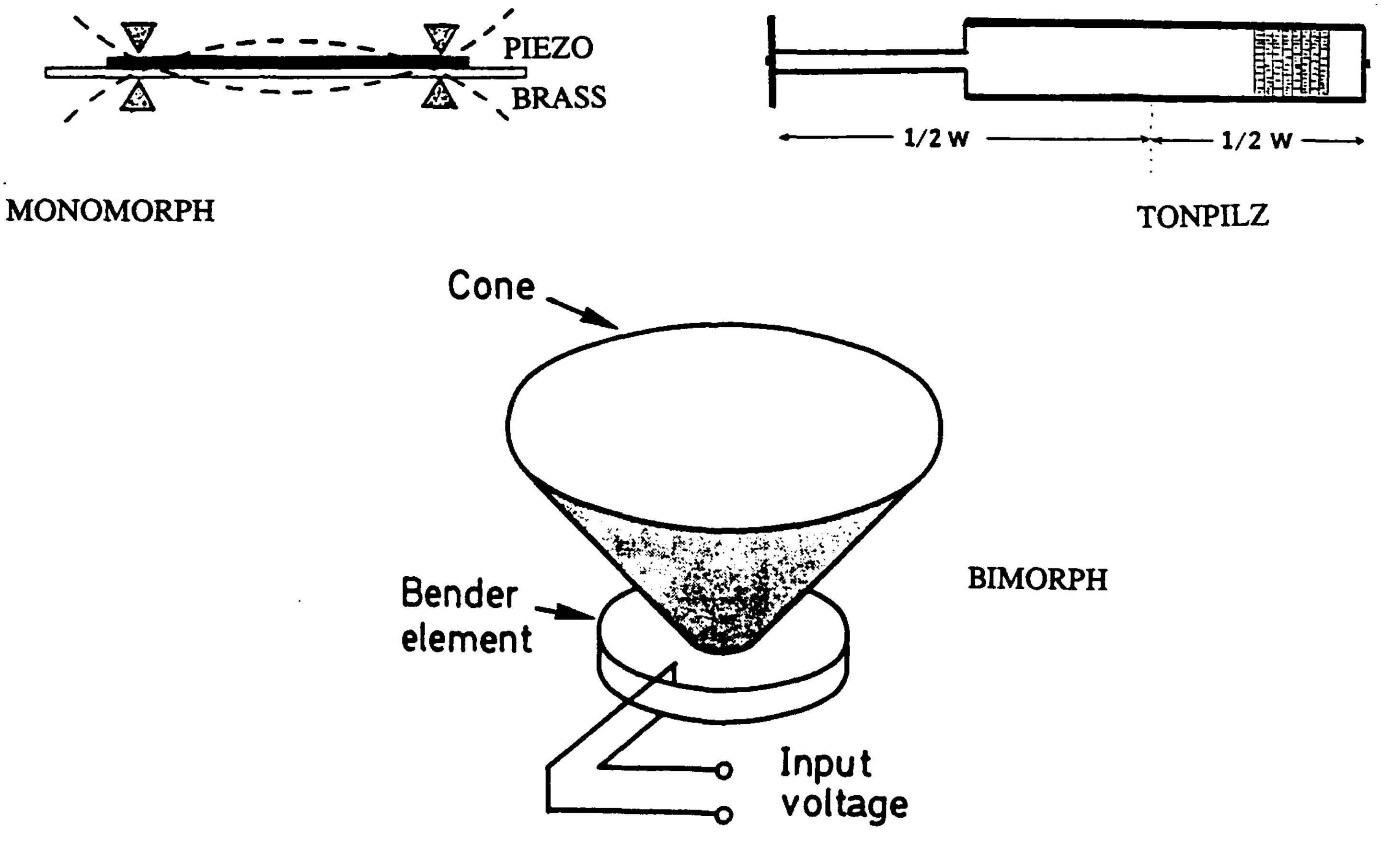

Fig. 2 Piezoelectric drivers used in thermoacoustic refrigerator

The monomorph driver was 1" diameter and it consisted of a disc of lead zirconate titanate, type EC66, attached to a brass plate. It was manufactured by EDO Corporation, Salt Lake City. Its mechanical Q was 80 and it was built to resonate in its half-wave mode at a resonant frequency of ~2.5 kHz. The sensitivity of this driver was:

70dB at 1 ft for 1V rms across it.

The 0dB reference corresponds to 2×10^{-5} Pa. It produced a displacement of 0.010 mils per volt of excitation across it. This driver was very useful in the early stages of the project as it demonstrated cooling which produced ΔT of ~5°C. It was abandoned as it lacked acoustic power.

The Bimorph Driver was developed by Motorola and it is a standard commercial item used in most piezoelectric tweeters. The element consists of a pair of piezoelectric plates in parallel working against each other; this causes a powerful bending action. A cone is attached to this element for impedance matching and this provides an increase in sound level by 10dB. The depth of the cone is 9.2 mm and its diameter is 3.6 cm. Its sensitivity is:

110dB at the surface of cone for 0.1 volt across it.

Although rated for a peak power of 75 watts, it can accept continuously 25 watts, at best. Continuous operation at slightly higher power levels leads to device failure after about 10-15 minutes. Its capacitance is 147.5 nF. Sensitivity varies in a batch of drivers by as much as 10dB. These transducers are basically broad-band ranging from about 2 kHz to 20 kHz. This type of driver is attractive for this project as it forms a compact unit with the rest of the refrigerator. Results with this driver gave ΔT of 6°C to 12°C.

The Tonpilz driver was developed for high intensity sound fields. It is usually used in the ultrasonic range for underwater applications and for ultrasound soldering irons. It essentially consists of:

--stack of piezoelectric drivers, attached in parallel electrical connections, resonated with masses at each end.
--transmission line with mechanical transformers
--vibrating plate to produce sound field

The stack consists of 16 piezoelectric transducers, PZT type EC66, EDO Corp., poled to produce longitudinal displacements. They were 2.9 cm diameter with a 0.9 cm diameter hole through the center. Parallel electrical connections were made with copper shims. Tensioning was provided by a steel bolt which squeezed the stack between aluminum resonators. The capacitance of the stack was 24 nFarads and it resonated at 5020 Hz. Matching to the load was achieved by means of an aluminum transmission line, ¼ wavelength long and 3.8 cm diameter, tapering to a thinner transmission line 1-2 cm diameter also ¼ wavelength long. Such a stepped transmission line provided mechanical gain of 10dB. Finally this unit drove a thin aluminum plate 3.1 cm diameter and 1 mm thick. The whole assembly was tuned to 5kHz with a Q of ~280. Its sensitivity on resonance was:

110dB at 0.1V across the stack.

It was capable of being driven at much higher voltages than the bimorph. Fig. 3 shows the set-up.

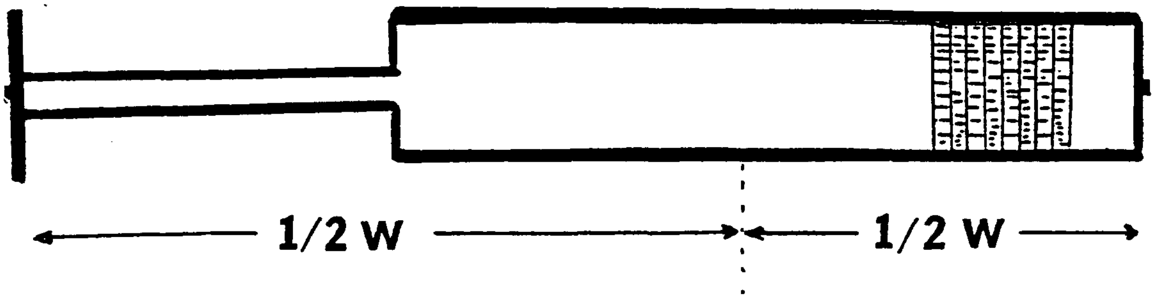

Fig. 3 Tonpilz driver tuned to 5 kHz

Power for the driver was provided by a Realistic PA amplifier, model M9A-95, capable of delivering 100 watts to a high impedance load (the driver impedance was ~100 ohms on resonance). We had problems with this unit at high levels because of:

--excitations of higher modes

-- difficulties in maintaining tuning because of the high Q of system

We could achieve at times sound levels of 170dB.

B. Stack

Two types of stacks were tried. They were:

--conventional stack developed by Hofler[6] consisting of mylar sheet rolled into a pancake structure with each sheet separated by nylon fishing line.

--random fibrous material of a plastic composite designed for swamp coolers because of large surface area.

Since the radian wavelength with helium gas for our refrigerator is ~3cm, we used a stack which is 0.8cm long. This is in fact 10 times shorter than the stack used by Hofler.

The mylar-fishing line stack consisted of 0.002" thick mylar with 0.005" fishing line spacers. This was dictated by a thermal penetration depth δ_k of ~0.03mm. Such stack had a well-defined geometry useful for calculations and design features but it was difficult to make good thermal contact to heat exchangers in a uniform fashion. Typical stack surface area was 690cm^2.

For this reason, we tried a random system as a stack, which although it did not have an optimum geometry, it made better thermal contact to the heat exchangers. The results with this stack were very encouraging.

C. Heat Exchangers

Four types of heat exchangers were studied, all made out of copper. At first the heat exchangers consisted of copper strips 0.8mm thick, 2.5mm long, soft-soldered to a brass holder, and ground flat. This was pushed against the stack. Then we used heat exchangers of fine copper wires, 0.25mm in diameter, spaced by 0.25mm, which were soft-soldered to a copper holder. Here as well, this was pushed against the stack. Finally we ended up with heat exchangers which were photo-lithographically patterned copper plates. This is shown in Fig. 4. The thickness was 0.25mm.

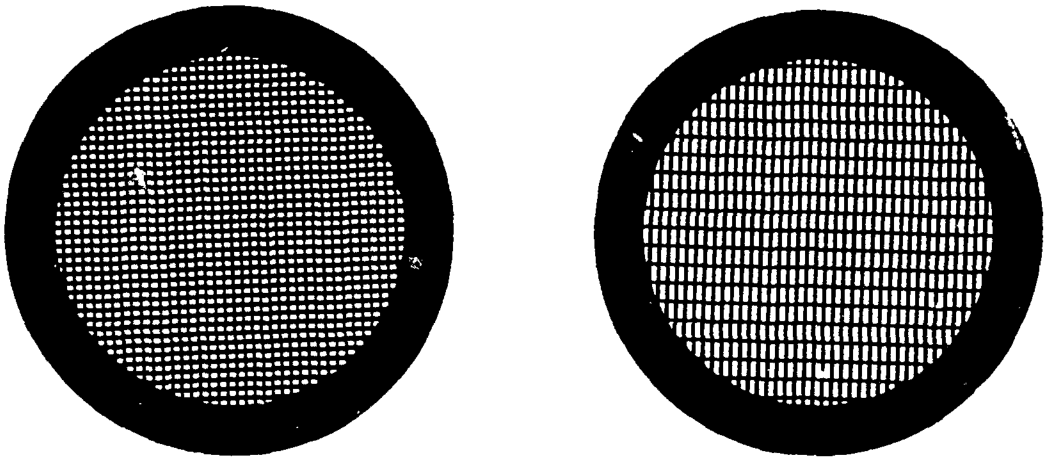

Fig. 4 Photolithographically prepared copper heat exchangers

The thermal contact between heat exchangers and the stack was one of the most serious problems in developing the high frequency refrigerator. In the 500Hz refrigerator thermal contact was made by relying on the acoustic displacement amplitude $u_1/\omega = 0.25$cm; it is so large that heat from the stack to the exchanger is simply transferred for a reasonable contact between the two. In our case u_1/ω was much less than 0.1mm and this meant that if the heat exchanger and stack were not flat to this degree, there would be poor thermal contact between the two. For this reason the random spongy stack made better contact to the heat exchangers.

D. <u>Resonator</u>

The preliminary experiments used a ½ wave cylindrical resonator for its simplicity. Improvements were made by using a Helmholtz resonator as it had smaller losses and its dimensions were much smaller than a wavelength of sound excitation. Its resonant frequency is given by:

$$\omega_o = c\,(S/lV)^{1/2} \qquad (12)$$

where the symbols used are shown in Fig. 5.

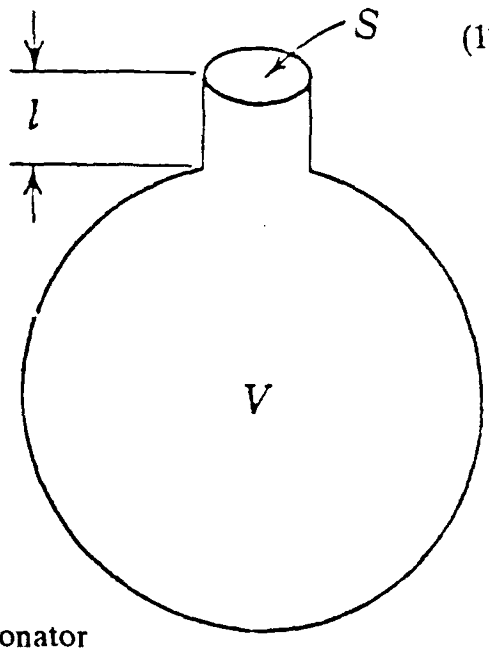

Fig. 5 Helmholtz resonator

The quality factor becomes

$$Q \approx 2\pi(c/\omega_o)l/S \tag{13}$$

where viscous losses are neglected and only the radiation losses are considered. Typical volume of resonator was 26.5 cc. Helium in this resonator was contained with pressures ranging from 10 to 20 atmospheres.

The resonator was fabricated out of stainless steel; its diameter was 3.75 cm. We also had resonators fabricated by casting a ping-pong ball into epoxy and then opening a port to the inside. By casting the ball in epoxy, the surrounding epoxy acted as a thermal insulator for the refrigerator. That was the only thermal isolation that was used in this project.

E. Other Details

Sound levels were determined by electret microphones and dB meters. Temperatures were measured by constantan-chromium thermocouples.

4. Results

A. Refrigerator Driven by Bimorph Driver

The driver in the refrigerator was a Motorola Piezoelectric Tweeter, KSN 1005, where the outside casing and the external exponential horn were removed. What was left was the bimorph and its cone load. Fig. 6 shows its broad-band frequency response.

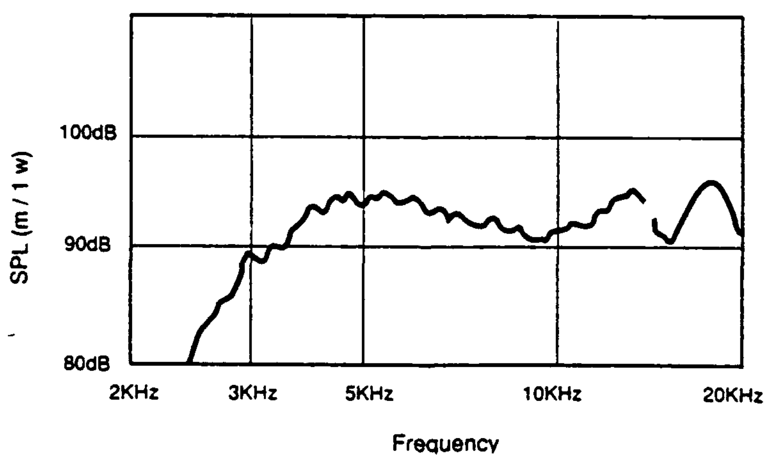

Fig. 6 Frequency response of Motorola Driver

Helium gas pressurized to 10 atmospheres was used as the working fluid.

For electrical power input of 45 watts to the driver a $\Delta p/p_o = 2 \times 10^{-3}$ was achieved at the position of the stack. For these conditions a rapid cooling was observed leading to a $\Delta T \sim 10-12$°C across the stack. There was no thermal isolation for the stack nor for the heat exchangers. No effort was taken to reduce stray heat influences as we wanted to see how well the refrigerator would perform on its own.

Fig. 7 shows the temperature difference developed across the stack for various electric power levels to the driver.

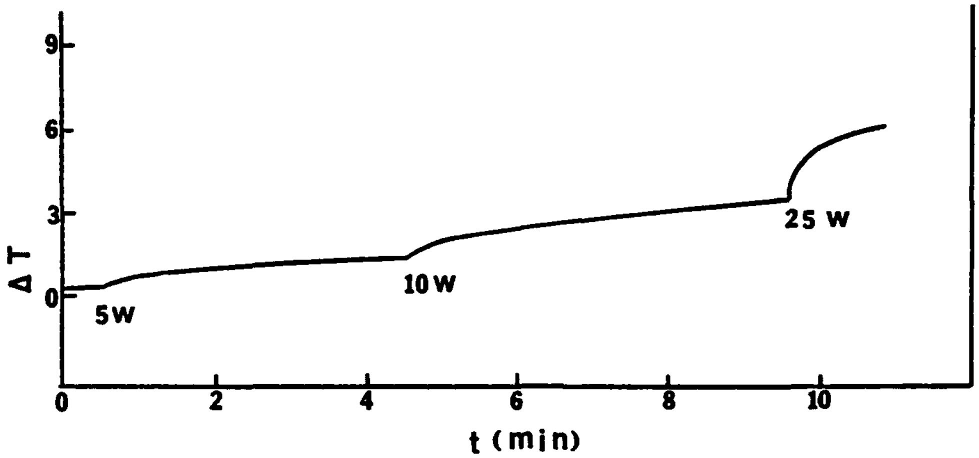

Fig. 7 Temperature differences developed across stack for various power inputs to driver

When the frequency was changed to off resonance there was no cooling. Slight detuning of resonance caused the system to stop cooling. The resonance condition was followed continuously by adjusting the tuning manually as the refrigerator cooled.

Fig. 8 shows the cooling curve across the stack for an electrical power input of 40 watts to the driver.

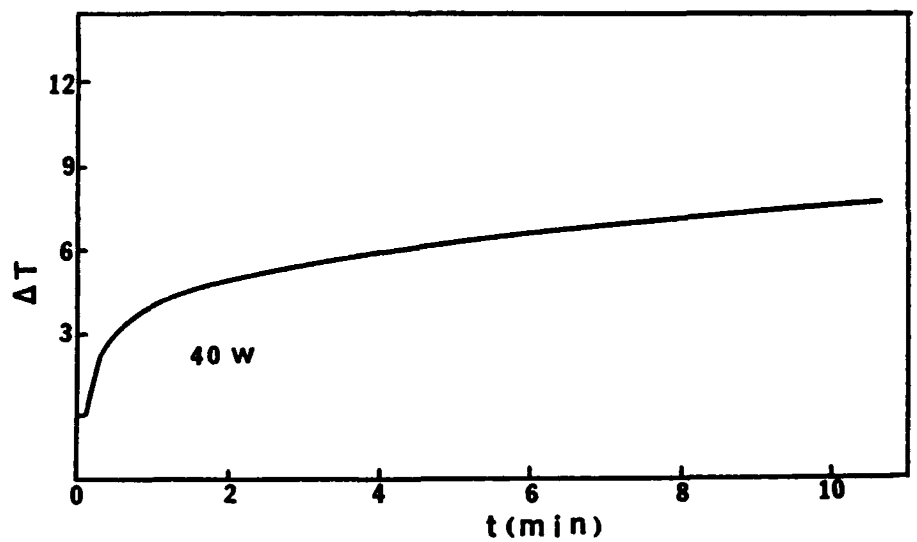

Fig. 8 Cooling rate of refrigerator at 40 watts of electrical power to driver

It is interesting to compare our results to the achievements of the Los Alamos Group of Wheatley and collaborators, helium is the working fluid.

Los Alamos Group	Present results
$\Delta p/p_o = 10^{-2} \rightarrow \Delta T = 5°C$	$\Delta p/p_o = 2 \times 10^{-3} \rightarrow \Delta T = 10°C$
$\Delta p/p_o = 4 \times 10^{-2} \rightarrow \Delta T = 100°C$	
$f = 500$ Hz	$f = 5{,}000$ Hz

The cooling performance was very much dependent on the quality of the driver. In a batch there was a variation by about 10dB between the different drivers. At too high power levels the drivers would get damaged.

The maximum sound level produced by this type of driver was 155dB to 160dB at the stack. Further improvements require more sound. Since we were at the limit of what was available in bimorph drivers for our application, we decided to try another type of driver, the Tonpilz Driver.

B. Refrigerator Driven by Tonpilz Driver

This is a resonant stack of 12 piezoelectric plates glued together in pairs, and loaded by a tensioning screw to an aluminum bar at one end and a stainless steel bar at the other end.[7,8] Fig. 9 shows the Tonpilz driver as used here.

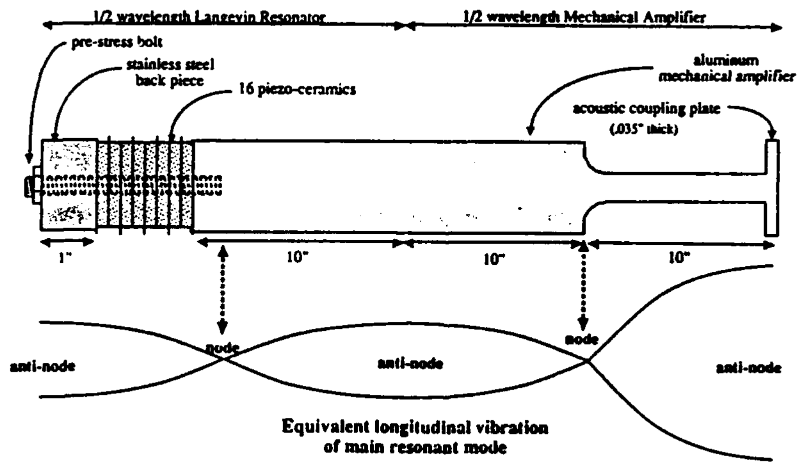

Fig. 9 Tonpilz Driver

It is a system resonant at 5,000 Hz and hence it is quite long, being 86.4 cm for the driver and the mechanical impedance match. This unit was contained in a chamber which had the pressurized helium gas; it was connected to the acoustic resonator. To reduce higher mode excitations a brass ring was mechanically clamped to the ½ wave resonant driver at its displacement node for the fundamental mode. Actually that point was not a perfect node, it was a displacement minimum, as there was a mixture of standing wave and travelling wave components.

With this driver cooling was also observed. Here as well, the only thermal isolation was the fiberglass around the Helmholtz resonator.

It was difficult to produce temperature differences larger than $\Delta T = 7\frac{1}{2}°C$; this was achieved for a sound level of 165dB. When the acoustic level was raised higher, secondary nodes in the driver got excited and the power input was siphoned into these modes rather than to remain in the main resonance. Although this driver could put out large sound levels, 170dB being the maximum that we obtained, higher modes always got mixed in. This also led to the detuning of the driver. Being a high Q system, it was difficult to stay on resonance in the presence of the other exicted modes. Automatic frequency control would have been useful.[9,10]

We measured the performance of the refrigerator as a function of static pressure on the helium gas. The largest ΔT was attined at a static pressure of 10 atmospheres; the variation with pressure was not large, being a few percent between 10 and 20 atmospheres.

In principle this driver has the potential for producing very high levels of acoustic power: its efficiency can approach 70 to 90% and it can handle high voltage levels. Unfortunately non-linearities of the system led to problems in controlling the oscillations of this driver.

This driver had another problem in that is was very long (86.4 cm) which was contrary to the goals of this project, miniaturization. Its use here was to show the principles but it was not meant as a working element of the miniature refrigerator.

C. Refrigerator Performance

Although the sound levels were low, the mini-refrigerator showed that cooling occurred by the thermoacoustic effect. Modest relative sound levels with $\Delta p/p_o \sim 2\times10^{-3}$ caused a temperature difference to be developed across the stack ΔT 10–12 °C, and this is without thermal isolation from the surroundings. To verify this we tried an 8-ohm 85-watt electromagnetic tweeter driver. This led to a $\Delta T=20°C$, the hot heat exchanger being cooled by circulating water. The measured efficiency of this driver was 3.5%. For a maximum electrical input of 90 watts, to the driver, the sound field inside the refrigerator was 3.15 watts.

Fig. 10 shows a cooling curve when the sound intensity was on followed by a warm-up when the sound was off. This provides us with a method for assessing, roughly, the performance of the refrigerator.

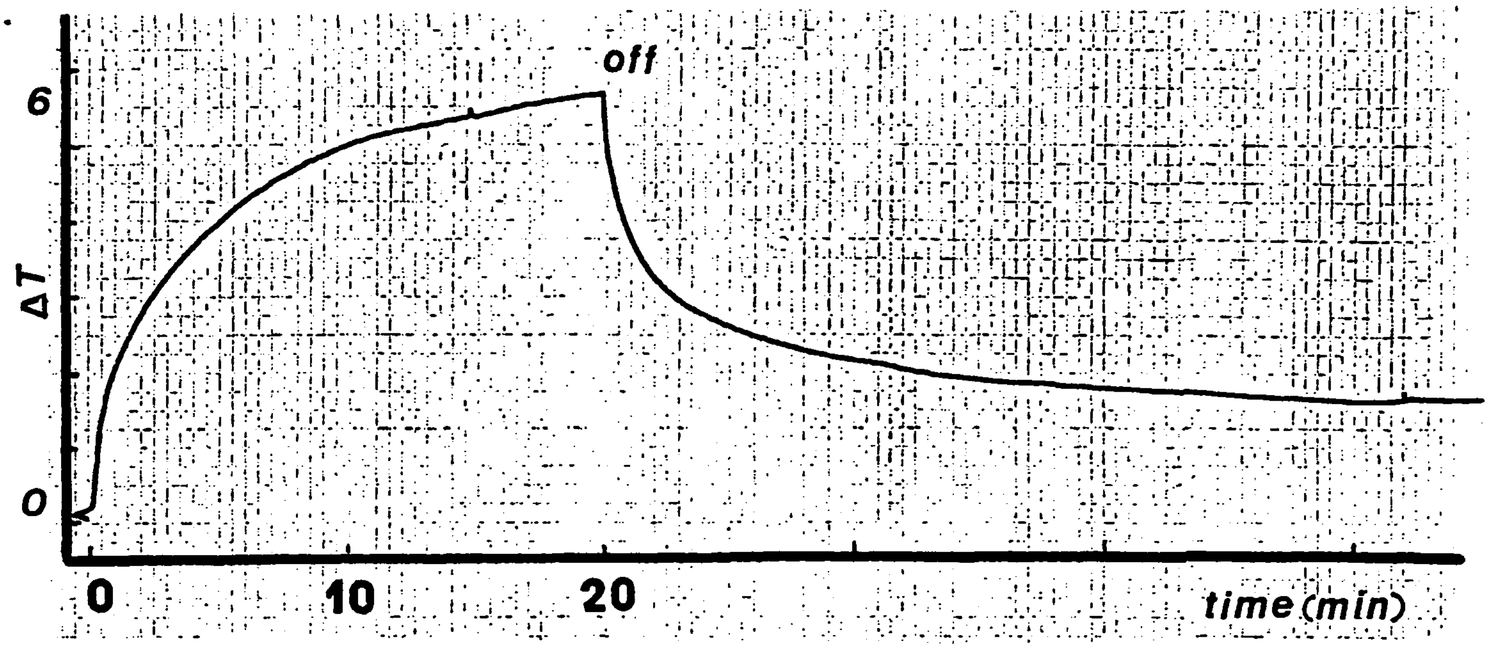

Fig. 10 Cooling of refrigerator when driven at 5,000 Hz at a level of 153dB followed by natural warming

From the rate of temperature rise, the rate of heat influx to the cold heat exchanger is calculated to be:

$$Q_{\text{heat leak}} = 0.22 \text{ watt}$$

In that case, from the cooling rate, the refrigerator power must have been approximately:

$$Q_{\text{refrigerator}} = 0.25 \text{ watt}$$

The sound level which produced such refrigeration was 153 dB and this corresponds to an intensity of 2×10^{-1} W/cm^2. This is 2 watts of acoustic power.

Some of the heat influx is from:

--across the stack, through the mylar
-- helium gas in stack
-- outside container of stack shorting out thermally the heat exchanger.
Had the stray head influx been 0, the temperature difference across the stack would have been ΔT=30°C instead of 6°C.

5. Discussion on High Frequency Operation

We have shown that a mini-refrigerator driven by a small piezoelectric driver at 5,000 Hz produces substantial cooling when operated with pressureized helium gas. The achieved temperature difference between the heat exchangers was 10–12°C without thermal isolation. In assessing the performance of the mini-refrigerator, it is instructive to compare it to the refrigerator of Wheatley, Hofler, and collaborators.

	500 Hz Refrigerator (Hofler et al)	5 kHz mini-Refrigerator
length of stack, Δx:	8 cm	0.8 cm
circumferential length of stack, Π	490 cm	432 cm
volume of Helmholtz resonator	1060 c.c.	26.5 c.c.
thickness of stack plates	0.08 mm	0.05 mm
spacing between stack plates	0.38 mm	0.12 mm
$\Delta p/p_0$	0.01 to 0.04	0.002
δ_k	0.1 mm	0.03 mm
∇T_{crit}	15°C/cm	150°C/cm
¼ wavelength	47 cm	4.7 cm
acoustic displacement u/ω	0.2 cm	0.002 cm
thermal isolation	vacuum and cold shield	none
acoustic driver	electromagnetic	piezoelectric

Some of the differences show why it is difficult to produce a mini-refrigerator. As the stack length Δx gets shorter, the intrinsic heat losses increase.

At this stage it is important to consider some of the important differences between the 500 Hz refrigerator and our refrigerator.

(a) Acoustic displacement amplitude u/ω: this is so small in our case that unless the heat exchanger touches well everywhere the stack, there will be poor heat transfer between them. For that reason a spongy random stack gave as good or better performance than a regular stack. Even for sound intensities 20dB or higher, that will still be a problem.
We propose incorporating the heat exchangers with stacks or to use the spongy glass wool type of stack.
Incidentally it takes about 400 cycles for a parcel of gas to travel from one end of the stack to the other end.

(b) Stack length Δx: it is short since it has to be much less than a radian wavelength. For a temperature difference of 100°C the heat influx by gas conduction would be 1 watt and for conduction through the stack it would be 0.4 watt. Hence for a $\Delta T=100$°C the cooling power would have to be larger. Other materials and gases would give smaller losses than 1.4 watt.

(c) Critical gradient: in a heat pump the maximum temperature gradient which can be developed is equal to ∇T_{crit} and it is given by:

$$\nabla T_{crit} = \left(\frac{\gamma-1}{T\beta}\right)\frac{T}{\lambda}\tan\left(\frac{x}{\lambda}\right) \tag{13}$$

where γ is the ratio of specific heats, β the thermal expansion coefficient, T is the mean temperature, x the distance to the pressure antinode, and λ the radian wavelength.
Because of the shorter wavelength, our critical gradient can be larger than for the 500 Hz refrigerator (but difficult to use since Δx becomes small). In fact we have observed that the positioning of the stack is critical to within a few millimeters in the resonating chamber.

(d) Efficiency: can be quite complex depending on how it is defined. First the efficiency of the refrigerator is given by:

$$\eta = \frac{\nabla T_{crit} \Delta x}{T} \tag{14}$$

which can be rewritten as:

$$\eta = \eta_c / \Gamma \tag{15}$$

where η_c is Carnot's efficiency, and Γ is the ratio of the temperature gradient across the stack divided by the critical gradient, $\nabla T/\nabla T_{crit}$.

The efficiency of the thermoacoustic refrigerator is less than that of a Carnot engine by the factor Γ. However, it is important to consider the efficiency of the driver; it has two contributions, the electrical efficency η_c and the mechanical efficiency η_m. In fact the driver efficiency can be written as:

$$\eta_{Dr} = \eta_e \, \eta_m \tag{16}$$

and this depends on the load. For piezoelectric drivers, the bimorph type or the Tonpilz type, the overall driver efficiency can be between 50 to 90%. This is larger than for an electromagnetic driver. Moreover, the weight of a driver like the bimorph is substantially lower than that of an electromagnetic driver where the magnet is the major contribution.

(e) Power density: this deals with the power per unit volume of the engine, including the resonator. Swift has derived an expression given as:

$$\frac{P}{V} \sim \frac{f}{2} \frac{T\beta P_1^2}{\rho_m a^2} \tag{17}$$

where f is the operating frequency, a the speed of sound, and P_1 the pressure amplitude of the sound. Eq. 17 shows that operation at high frequency f leads to a larger power density. This is not unexpected since at high frequencies the volume is smaller , and the cooling power also goes with the frequency. A benefit of high frequency operation is a larger power density.

(f) Viscous effects: influence the heat flow down the stack and the work flow. The viscous penetration depth depends on frequency and is given by:

$$\delta_v = (2\nu/\omega)^{1/2} \tag{18}$$

with the kinematic viscosity ν given by μ/ρ_m where μ is the viscosity. Viscosity prevents a layer ρ_v of gas next to the stack from moving acoustically and from transporting heat. Also it leads to lost energy from the sound field due to viscous drag on the plate. This layer decreases in thickness as the frequency is increased and is smalle rin our case than for the 500 Hz refrigerator.
Viscous losses were responsible for the reduction of the Q of the acoustic reonanto. The combination of stack and had exchangers reduced the Q by a factor of 2 to 3.

(g) Scaling: the refrigerator developed here was scaled down, geometrically, from the 500 Hz refrigerator of Wheatley and collaborators for operation at 5,00 Hz. There are some differences since our unit uses a piezoelectric driver which is lighter than the electromagnetic driver by orders of magnitude. In fact, this makes our unit more compact and more efficient. The limitation in our results has been the low power levels that the driver could deliver. Apart from that the performance was as expected. We did not expect a larger ΔT than what we obtained for the driver sound levels that we used.
By scaling down dimensions of the refrigerator, many possibilities exist. first, much higher static pressures can be used so as to better match the drive to the working fluid. Second, precision components can be fabricated by lithography. Third, the radiation losses areminimized as the dimensions of the device are reduced. The demonstration of scaling is an important element of this research as it shows that other refrigerators can be built along the modesl used here.
The mini-refrigerator developed here can be used for space applications[12], where electronic components need to be refrigerated and high-reliability performance is necessary.

6. References

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